Four-dimensional Riemannian manifolds with circulant structures and skew-circulant structures

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The geometry of Riemannian (pseudo-Riemannian) manifolds with additional structures, whose square is minus or plus the identity, has been systematically studied by many mathematicians. A modern part of these classes of manifolds are the Riemannian almost product manifolds and the almost Hermitian manifolds.

The circulant matrices and the skew-circulant matrices are Toeplitz matrices whose properties are well studied. The set of the invertible circulant (skew-circulant) matrices forms a group under the matrix multiplication. Such matrices have applications to geometry, linear codes, graph theory, and vibration analysis.

In this research, we study the geometric properties of a 4-dimensional Riemannian manifold equipped with an additional tensor structure, whose component matrix is circulant and its fourth power is the identity. Furthermore, this structure is compatible with the metric such that an isometry is induced in every tangent space. Every manifold of the considered type is associated with a Riemannian almost product manifold.

We also consider a 4-dimensional Riemannian manifold equipped with an additional tensor structure, whose fourth power is minus the identity. The component matrix of this structure is a skew-circulant matrix. We suppose the structure is an isometry with respect to the metric. Such a manifold is associated with an almost Hermitian manifold.

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