Anisotropic extensions of the multilinear spectral problem and their applications

Abstract. Among the promising estensions of the Riemannian framework, the Finsler extension proves its usefulness in numerous fields, like Biology, Physics, GTR, Nanotechnology and Geometry of Big Data. We describe the Finsler structures and several of their notable extensions and illustrate the theory by a brief account on their GTR applications. Further, we give the basic notions on spectra and critical values for tensors, and on Tucker-type decompositions. We provide several applications of Finsler models which emerge from Langmuir-Blodgett Nanotechnology and from Oncology. The Tucker decomposition, a powerful data structuring analysis tool, is exemplified for these models via Candecomp and HO-SVD, providing a relevant insight in the geometric underlying structures.

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