Veštačke neuronske mreže

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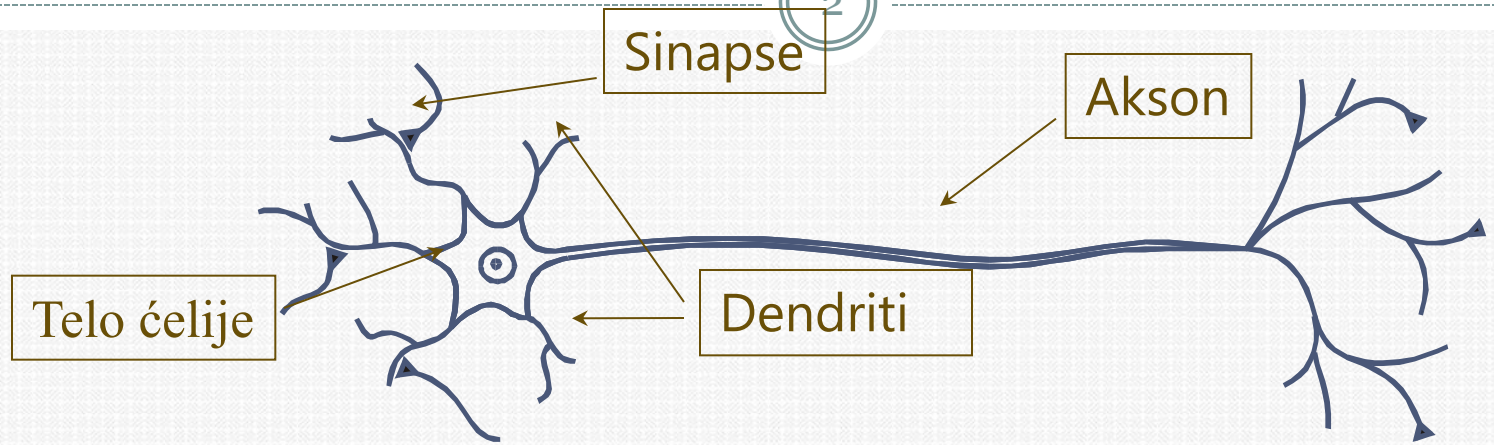
I Biološka inspiracija



- Ljudski mozak ima 80 milijardi neurona.
- Nervne ćelije prosleđuju signale desetinama hiljada drugih nervnih ćelija.

- Električni impuls koji se prostire duž aksona okida oslobadjanje neurotransmitera u sinaptičku pukotinu
- Neurotransmiter podstiče stvaranje električnog signala u postsinaptičkom neuronu (sinapsa je ekscitatorna) ili sprečava stvaranje električnog signala u postsinaptičkom neuronu (sinapsa je inhibitorna).
- Ukupan uticaj ekscitatornih ili inhibitornih signala može da proizvede električni signal u postsinaptičkom neuronu.

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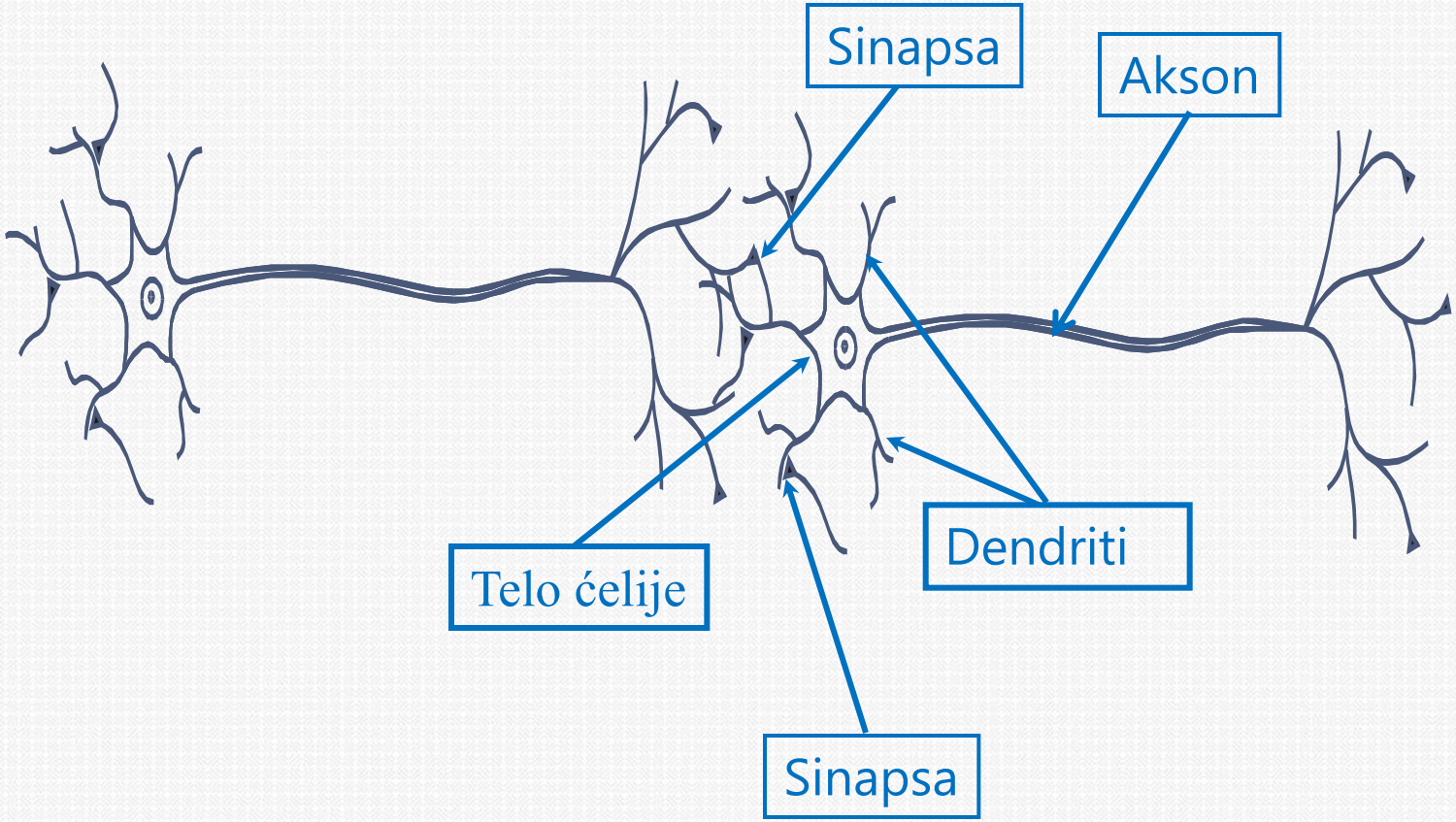


II Matematički modeli procesnih elemenata

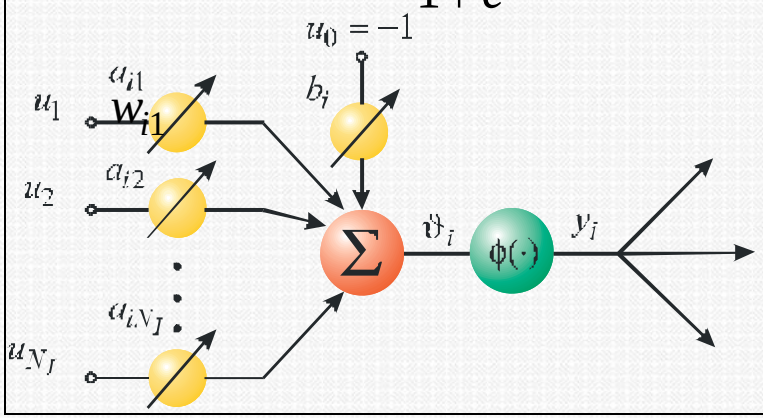
Model neurona(I)

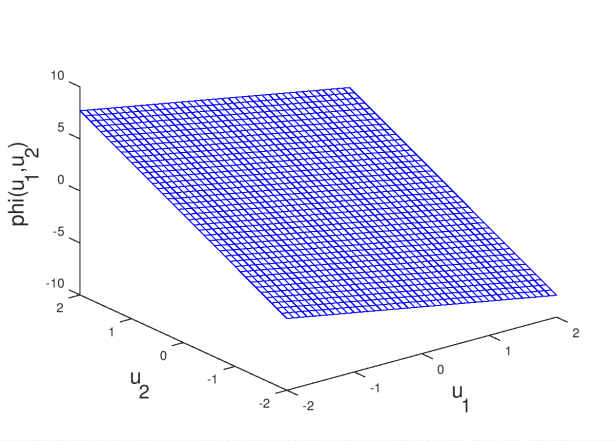
Potencijal:

$$a_i = \sum_{j=1}^{n_i} w_{ij} u_j - \theta_i = \sum_{j=1}^{n_i} w_{ij} u_j + w_{i0}$$

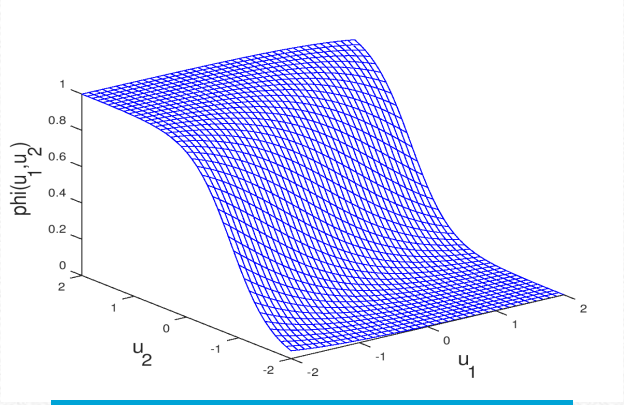
Aktivnost:

$$y_i = f(a_i) = \frac{1}{1+e^{-a_i}}$$





Potencijal



Aktivaciona funkcija

II Matematički modeli procesnih elemenata

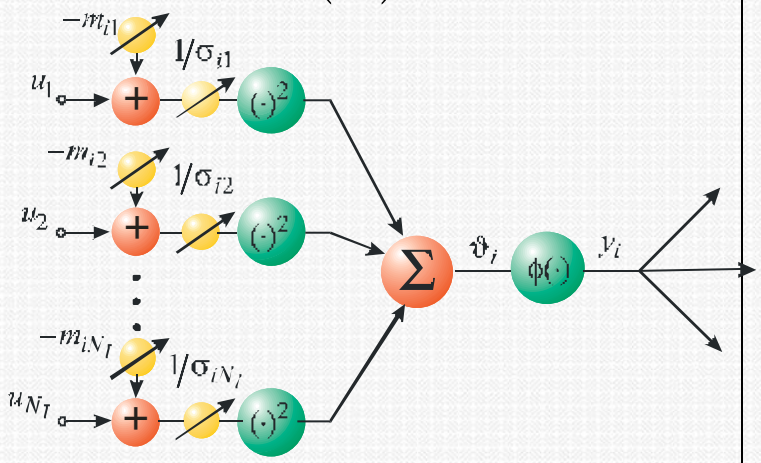
Model neurona(II)

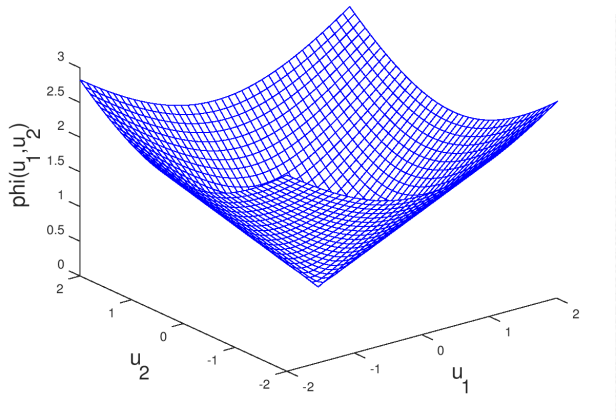
Potencijal:

$$a_i = \sqrt{\sum_{j=1}^{N_I} \frac{(u_j - m_{ij})^2}{\sigma_{ij}^2}}$$

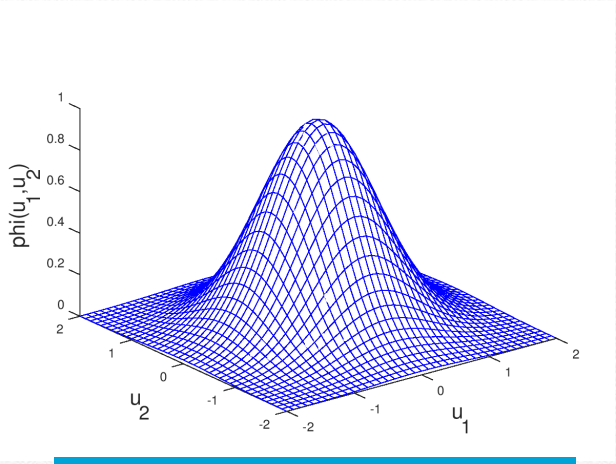
Aktivnost:

$$y_i = f(a_i) = e^{-a_i^2}$$



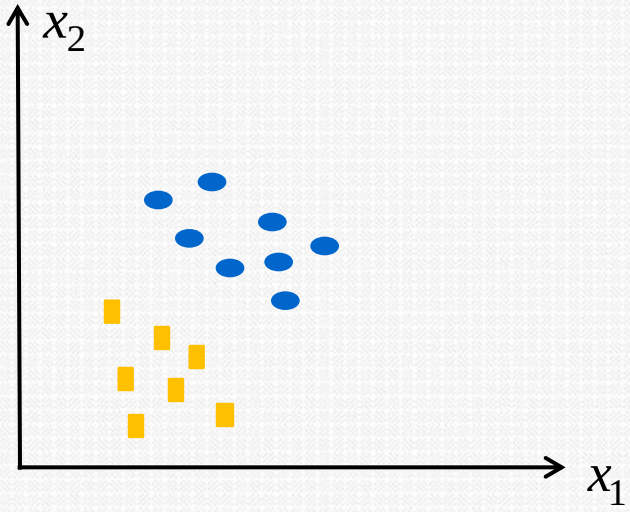


Potencijal



Aktivaciona funkcija

III Geometrija neurona



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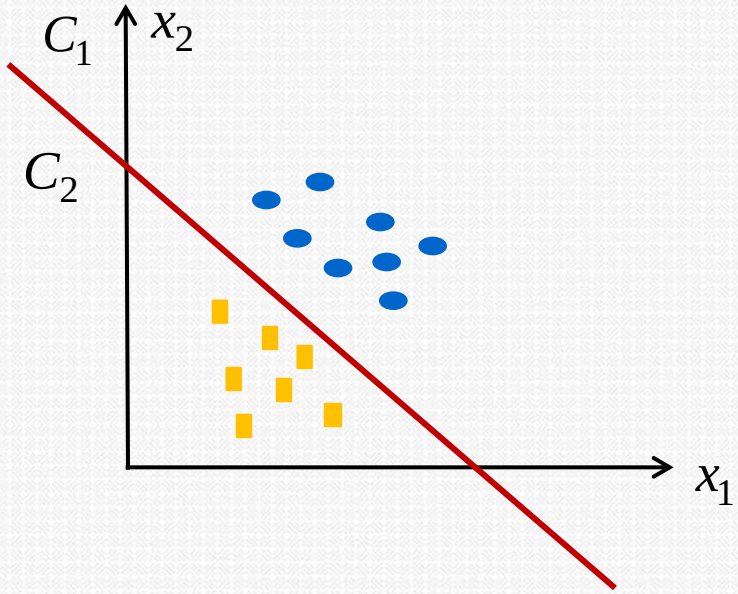

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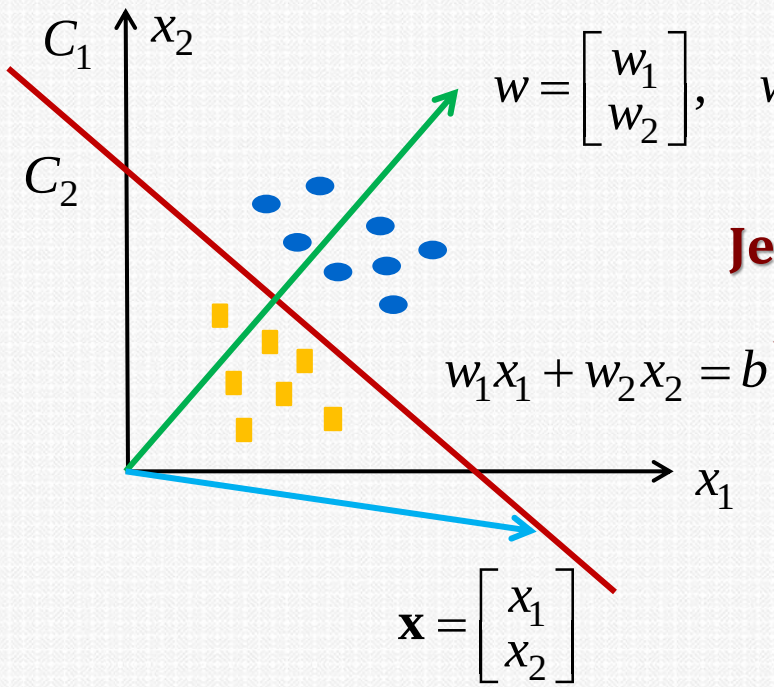

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad w \perp (w_1 x_1 + w_2 x_2 = b)$$

Jednačina prave koja razdvaja dve klase

$$w_1 x_1 + w_2 x_2 = b$$

$$w^T \mathbf{x} - b = 0, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \; w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

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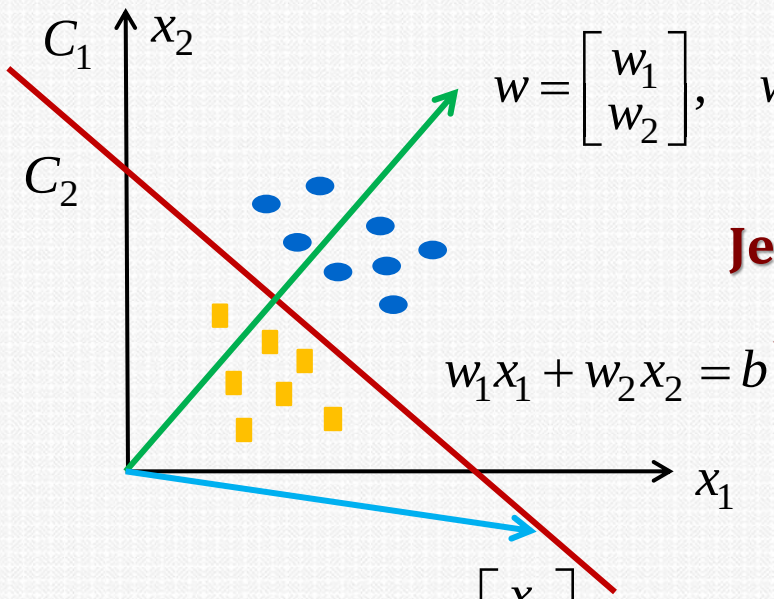

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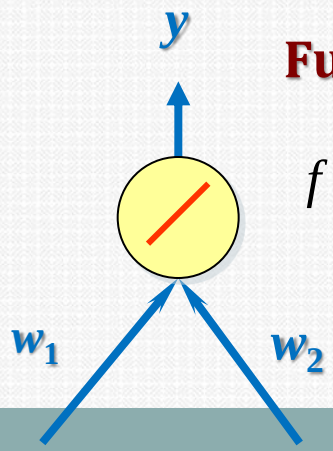
Funkcija klasifikatora

$$f(\mathbf{x}) = w^T \mathbf{x} - b, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



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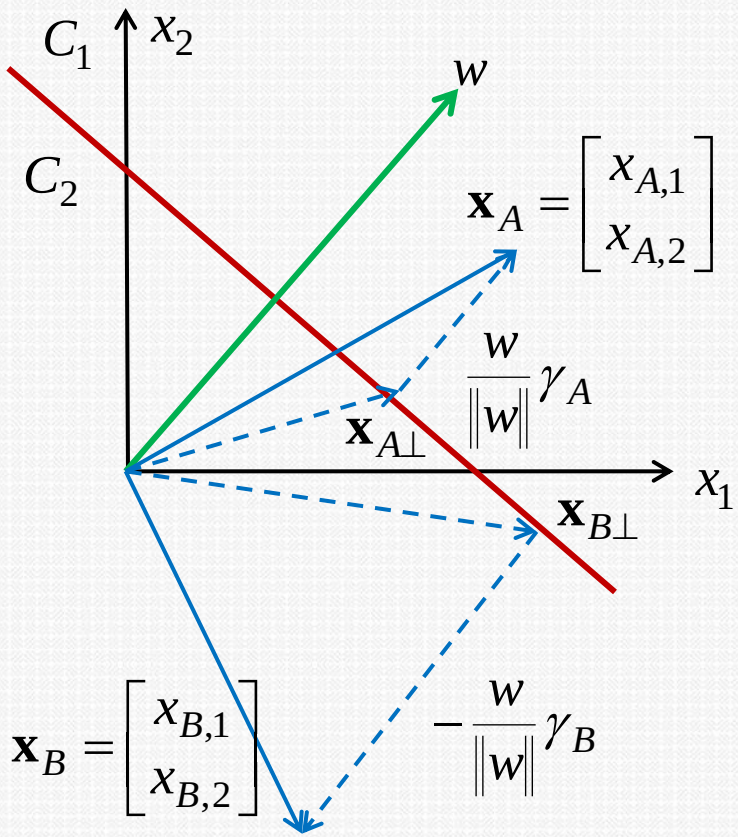


Odziv klasifikatora za ulazni uzorak predstavljen vektorom osobina **$\mathbf{x}_A$, koji pripada klasi C_1** je **pozitivan**.

$$f(\mathbf{x}_A) = w^T\mathbf{x}_A - b = w^T\left(\mathbf{x}_{A\perp} + \frac{w}{\|w\|}\gamma_A\right) - b$$

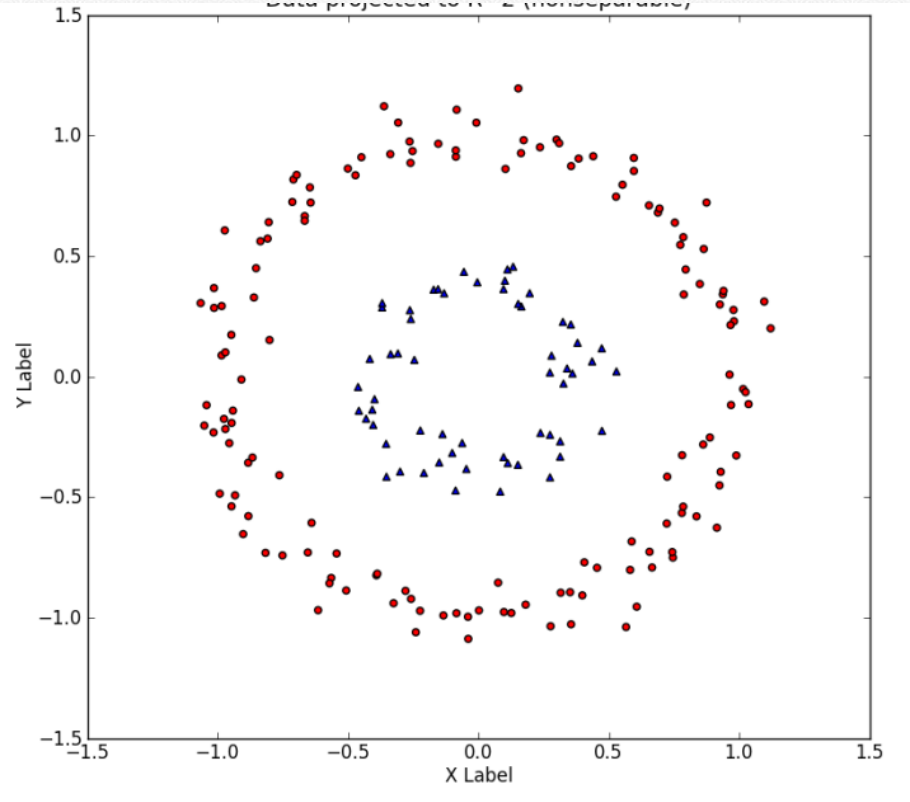
$$= w^T\mathbf{x}_{A\perp} + \frac{w^T w}{\|w\|}\gamma_A - b = \|w\|\gamma_A > 0$$

Odziv klasifikatora za ulazni uzorak predstavljen vektorom osobina **$\mathbf{x}_B$, koji pripada klasi C_2** je **negativan**.

$$f(\mathbf{x}_B) = w^T\mathbf{x}_B - b = w^T\left(\mathbf{x}_{B\perp} - \frac{w}{\|w\|}\gamma_B\right) - b$$

$$= w^T\mathbf{x}_{B\perp} - \frac{w^T w}{\|w\|}\gamma_B - b = -\|w\|\gamma_B < 0$$

III Geometrija neurona



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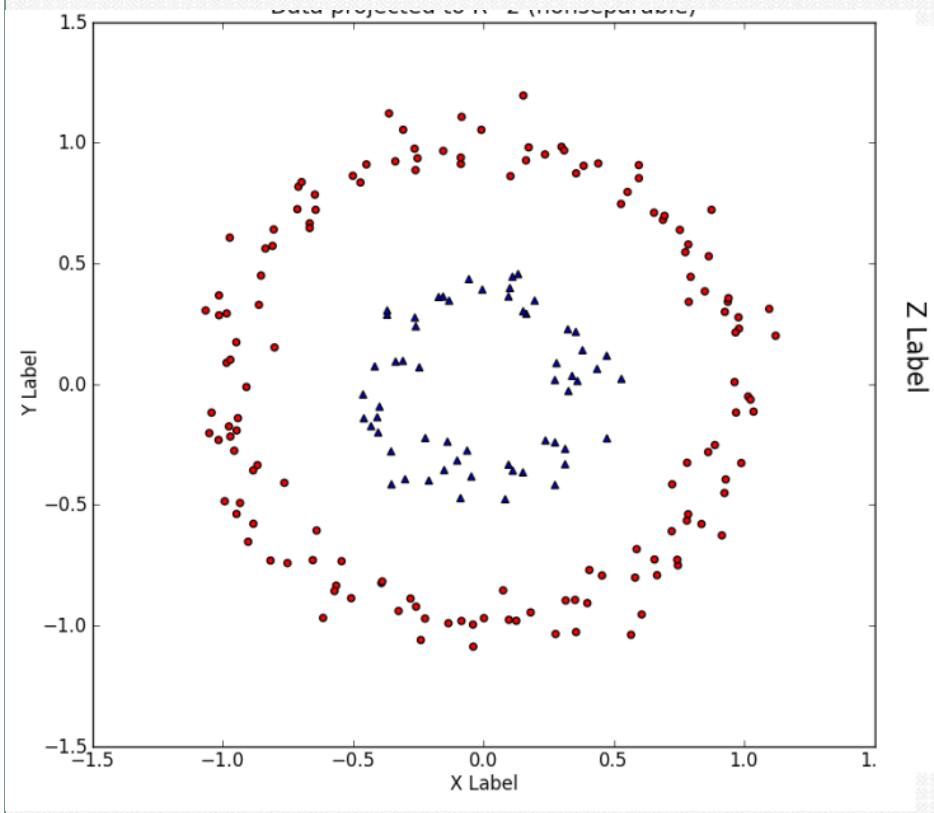
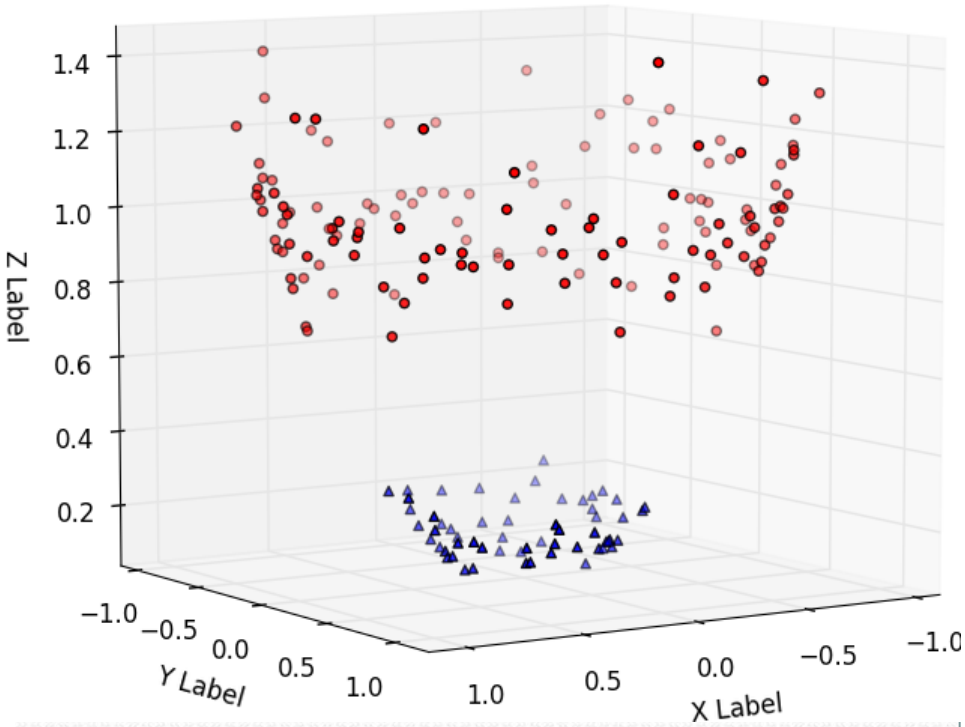
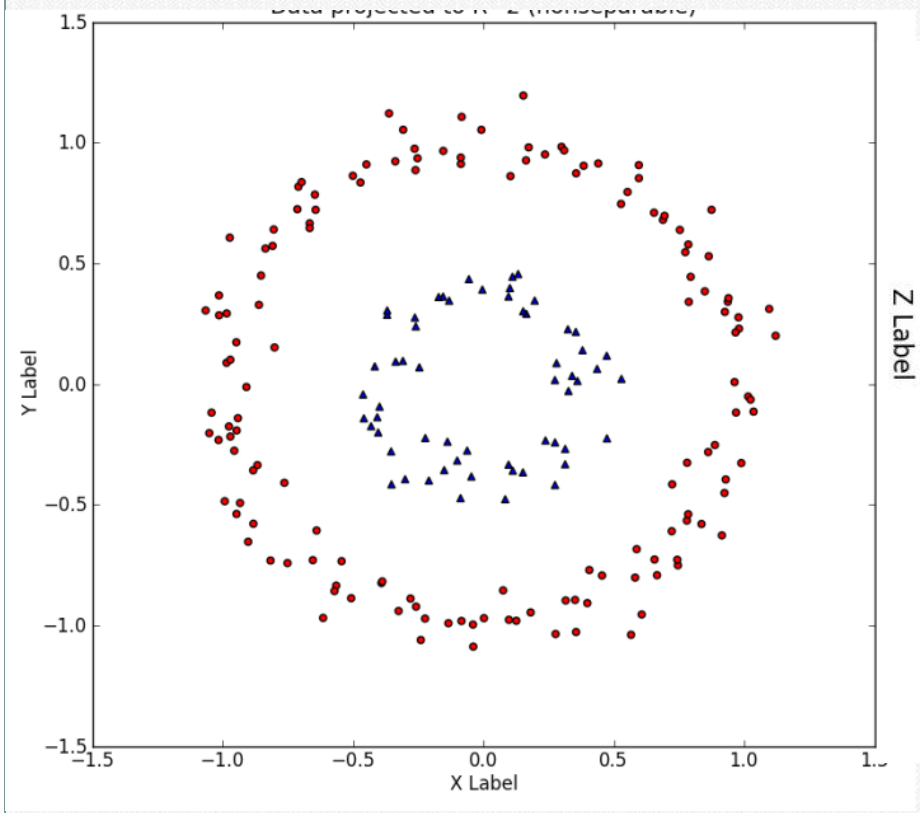
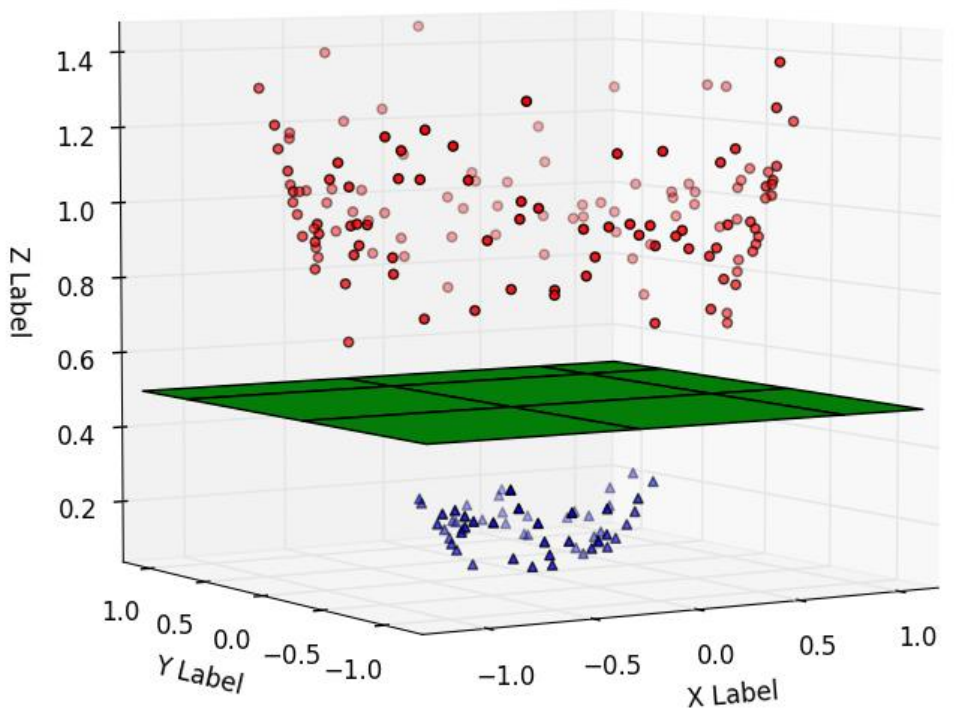




$$[x_1, x_2] \Rightarrow [x_1, x_2, x_1^2 + x_2^2]$$

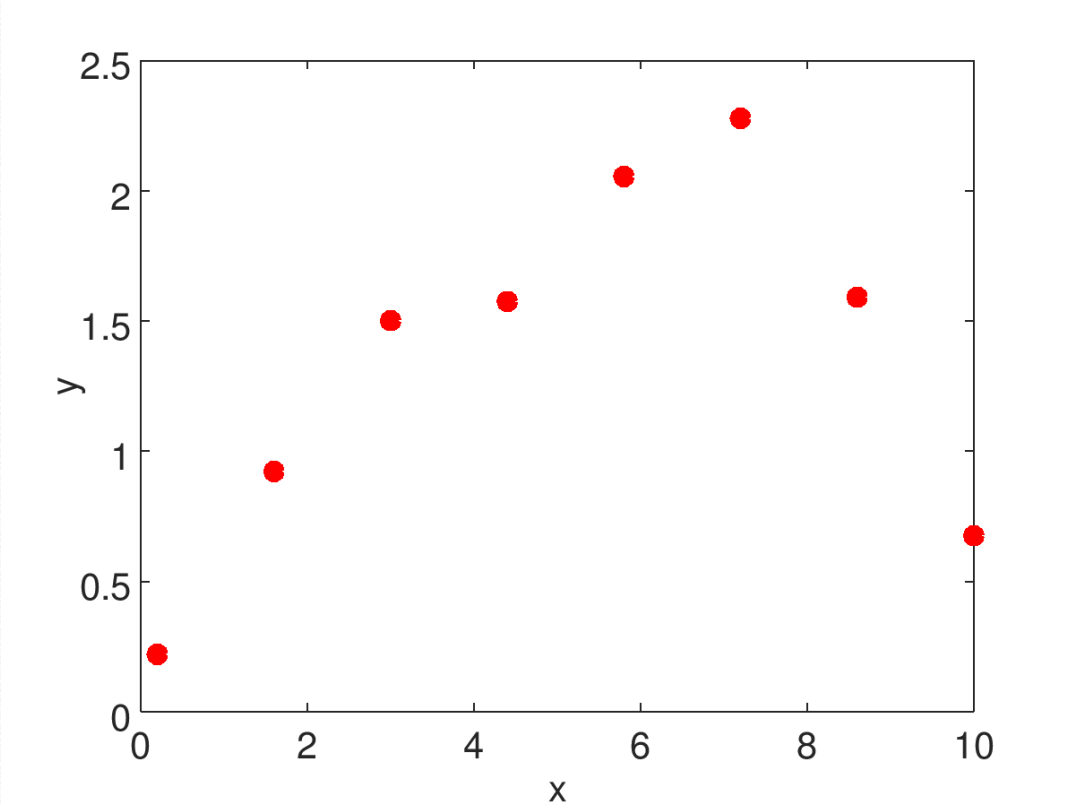
III Geometrija neurona





$$[x_1, x_2] \Rightarrow [x_1, x_2, x_1^2 + x_2^2]$$

IV Neuronska mreža



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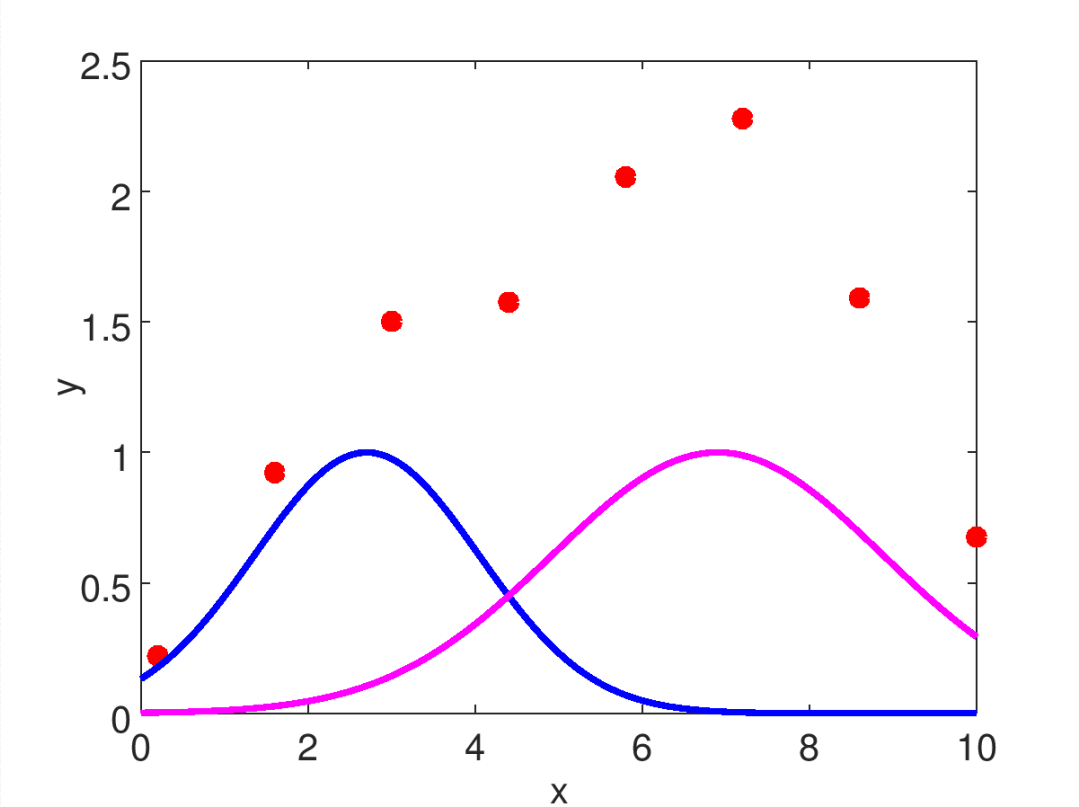

$$f_1(x) = e^{-\left(\frac{x-m_1}{\sigma_1}\right)}$$

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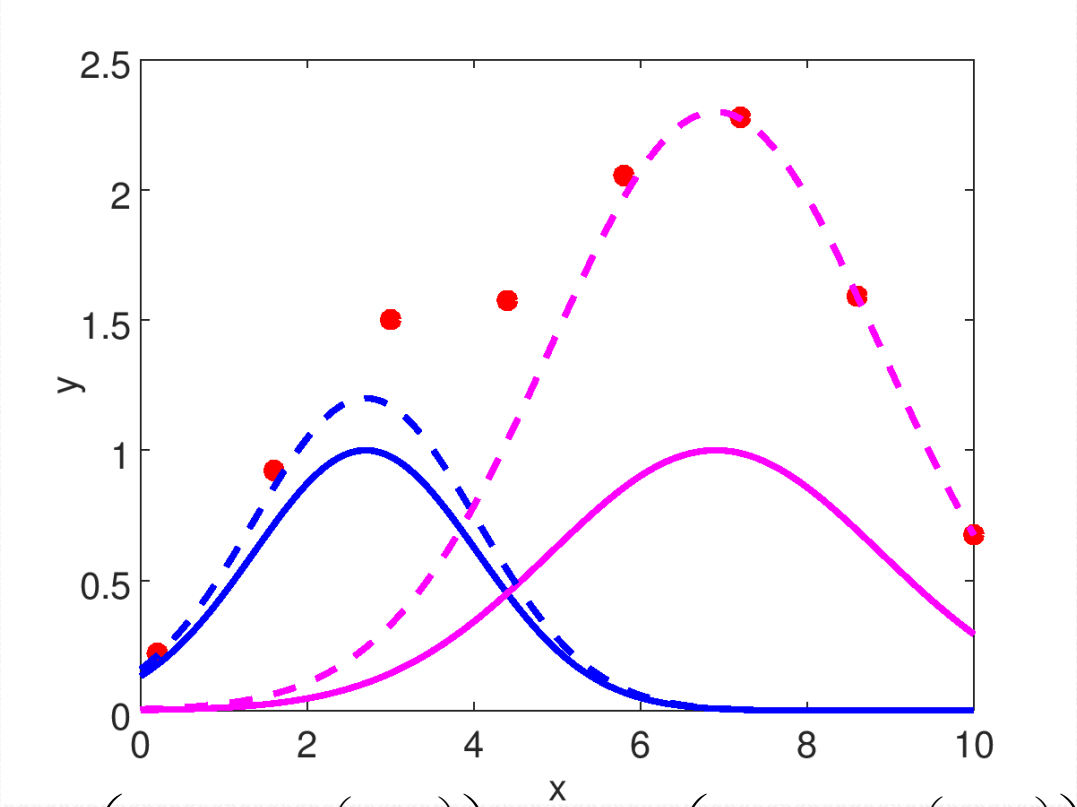
IV Neuronska mreža



$$w_1 * \left(f_1(x) = e^{-\left(\frac{x-m_1}{\sigma_1}\right)} \right) \quad w_2 * \left(f_2(x) = e^{-\left(\frac{x-m_2}{\sigma_2}\right)} \right)$$

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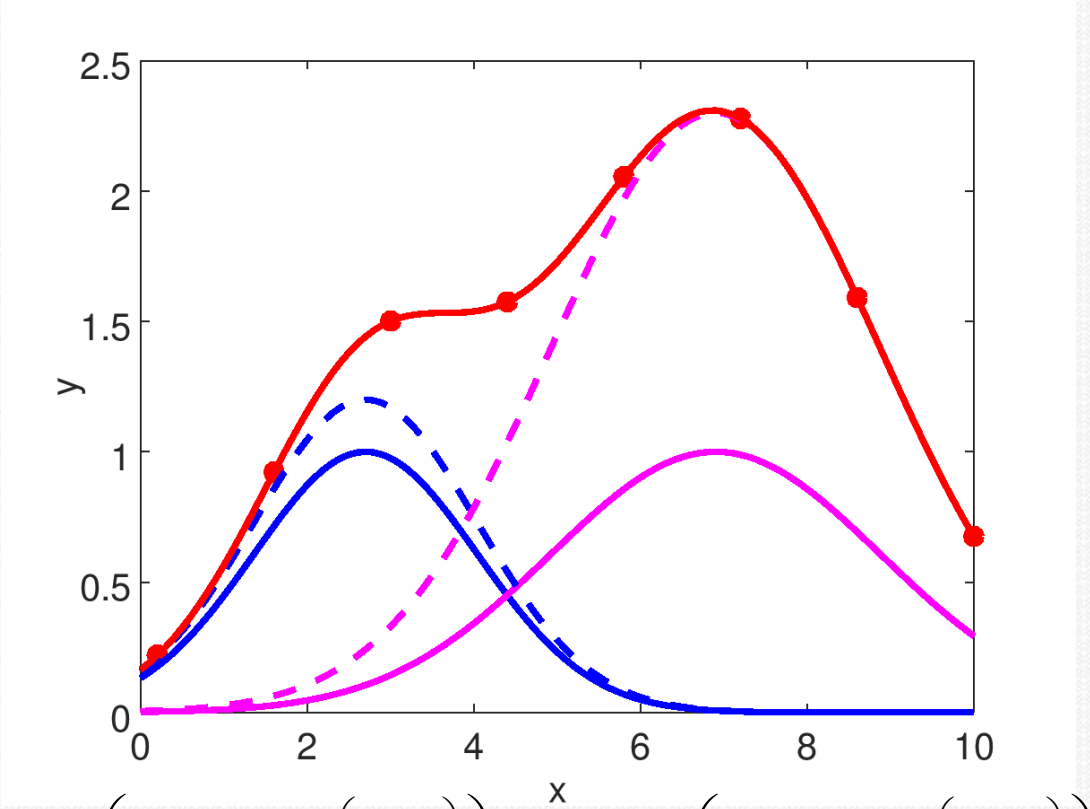

$$y = w_1 * \left(f_1(x) = e^{-\left(\frac{x-m_1}{\sigma_1}\right)} \right) + w_2 * \left(f_2(x) = e^{-\left(\frac{x-m_2}{\sigma_2}\right)} \right)$$

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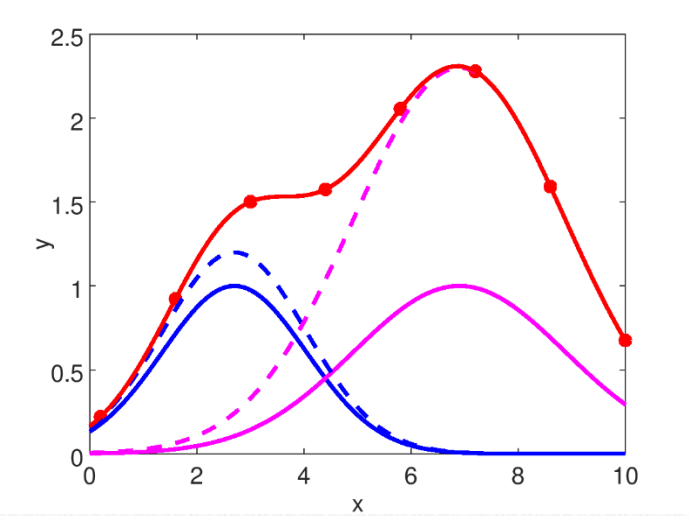
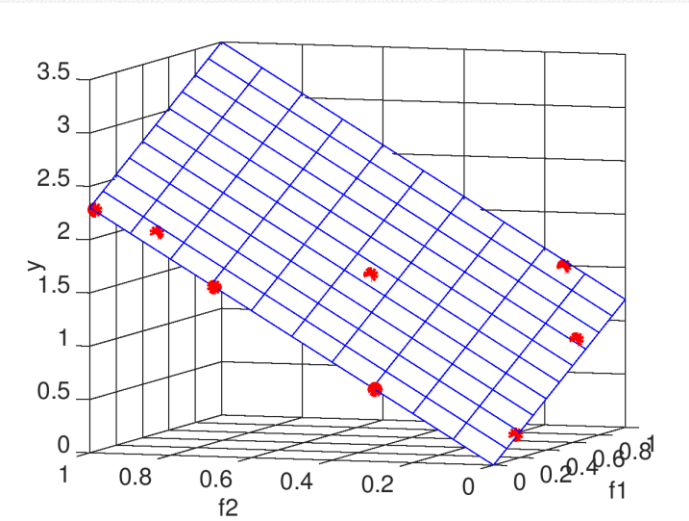
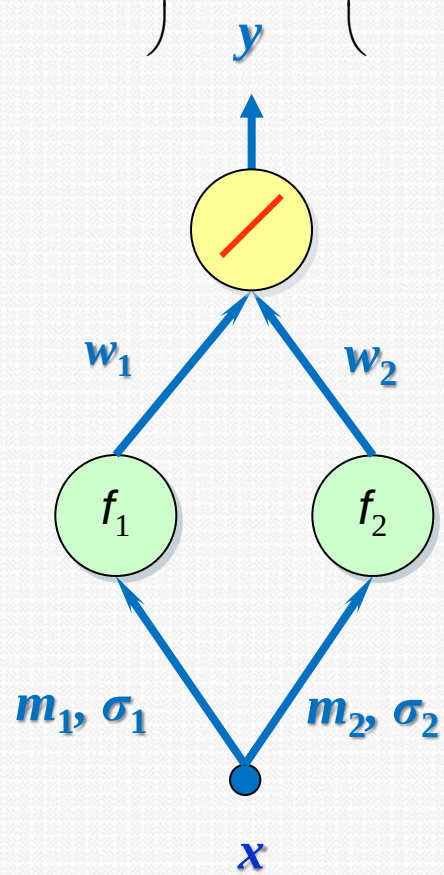




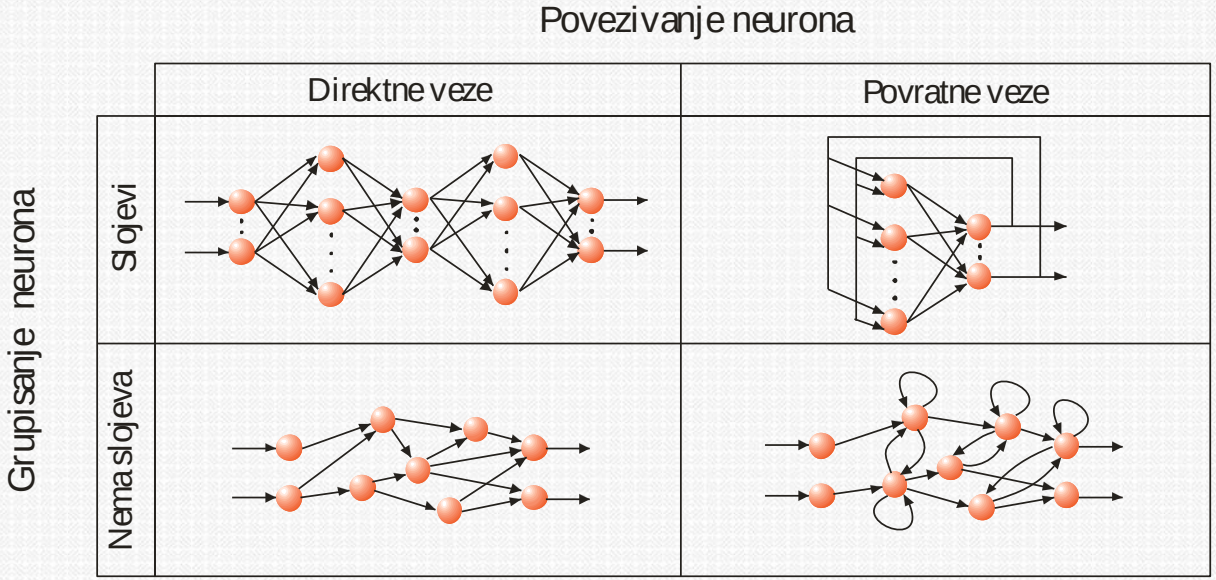
$$y = w_1 * \left(f_1(x) = e^{-\left(\frac{x-m_1}{\sigma_1}\right)} \right) + w_2 * \left(f_2(x) = e^{-\left(\frac{x-m_2}{\sigma_2}\right)} \right)$$



IV Neuronska mreža

Veštačke neuronske mreže se sastoje od (velikog) broja, medjusobno povezanih jednostavnih procesnih elemenata (veštačkih neurona).

Obrada podataka je distribuiriana i paralelna



V Recept za inaligentnu mašinu

- **Primeri na kojima treba da uči**
- **Model (memorija) u kome treba da čuva stečeno znanje**
- **Kriterijum koji kaže da li je to što je mašina naučila dovoljno dobro**
- **Algoritam učenja (optimizacioni metod), koja će menjati parametre (i strukturu) modela kako bi zadovoljio kriterijum**